Poor Man's Laser Scanner, a simple method of 3D cave surveying*

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We present our new 3D cave surveying technique and software, Poor Man's Laser Scanner (PMLS). Our method is based on splay shots performed with Beat Heeb's DistoX or similar laser distance meter equipped with compass and inclinometer. The sampled points should be distributed in all directions on all over the walls visible from the stations. Utilizing a robust and reliable surface reconstruction algorithm, our software interpolates the measured points with a watertight surface, which is free of self-intersections. We have found that even complicated geometric layouts can be recovered with good detail from as few as 50 to 150 splay shots per station. Our 3D models are more accurate and realistic than those generated by already existing, widespread cave mapping programs like Compass or Therion. We have surveyed Bányász cave, the deepest cave in Hungary, completely with impressive results. Using a tripod and a special DistoX firmware upgrade by Beat Heeb made on site work fast and convenient. The 3D model of the cave and further details about our method can be found on our website: cave3d.org.

Additional Key Words and Phrases Surface reconstruction, 3D cave survey

1 Introduction

The most widely used measuring device for cave surveying is Beat Heeb's DistoX [16] and DistoX2 [17], which is a laser distance meter equipped with compass and inclinometer. Using this device one can take several hundreds of splay shots from a station in a few minutes. Doing this systematically, we can make a sparse 3D survey of a cave. However, we can measure only about ten or twenty thousands of points in a day with a single DistoX, which is very few compared to point clouds obtained from Terrestrial Laser Scanners (TLS) [1] or GeoSlam's ZEB1/ZEB REVO handheld laser scanners [34]. In addition, the distribution of the sampled points can be extremely uneven. The question then arises: is it possible to acquire a 3D model of the cave based on such few and unevenly distributed measurements? Considering the price of TLS and ZEB devices and the fact that it is very complicated or even impossible to use TLS in narrow places, this problem is of great importance.

In this paper we give a first report on the Poor Man's Laser Scanner (PMLS), which is a new cave surveying technique and associated software based on splay shots performed with

the DistoX or DistoX2. We have developed a simple but yet robust and reliable surface reconstruction algorithm that interpolates the measured points with a watertight surface of good quality, which is free of self-intersections. Recent advances in 3D software technology improved the possibility of such software development significantly. Nowadays many software libraries and programs are available for manipulating and viewing 3D data effectively. These pieces of software comes from the field of 3D scanning, 3D medical imaging, and 3D animation. Building upon these tools we created a software solution for acquiring good quality, realistic 3D cave models from DistoX measurements with modest software development efforts. We have surveyed Hungary's deepest cave (273 m), the Bányász cave, which is about one kilometre long, and we think that the required on-site work is also reasonable. In one day we could survey 50-100 m long sections with a single disto. We compared one of our models with a dense pointcloud resulting from a thorough TLS survey. The vast majority of the TLS's points were closer to our model than 300 mm.

2 Related work

Let us take a look at the already existing methods that can provide 3D cave models. By conventional cave surveying, it is common to take splay shots in four directions with DistoX: left, right, up, and down (LRUD) in addition to the leg shots. Some widely used cave mapping programs (Compass [11], WinKarst, Therion [6]) are capable of making rough 3D models from centre-line and LRUD measurements. LRUD models are very inaccurate and not very realistic, but the survey is fast and cheap. Therion can also produce 3D models by combining passage outlines from digitized 2D maps and height data. In Hungary Joe Mészáros created some 3D models based on cross sections and centre-lines [23]. Both of these techniques result in unrealistic models. The problem with these approaches is that they try to recover the 3D layout from separate 2D and 1D information. This kind of divide and conquer strategy leads to poorly distributed sampling of 3D reality, because we can only build upon data points lying on specific cross section planes or some projection planes. In the case of Therion the information is furthermore distorted by the projection. Proper 3D reconstruction methods must use 3D data directly and treat all the three dimensions together.

Beside the techniques based on traditional cave mapping, there are solutions that can provide detailed, high quality

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models based on dense and accurate point clouds. Unfortunately the equipment has a price that definitely cannot be afforded by caving clubs. The terrestrial laser scanner is the equipment of professional 3D surveying [1]. TLS scanners are usually very accurate, even at a range of several hundred meters. On the other hand, it is impractical to use TLS in tight caves due to their size and fragility [18]. Since these devices must be mounted on a tripod, large cave chambers and wide passages are most suitable for surveying, where data can be captured from a modest number of stations [28], [21], [29], [2], [24], [27], [13], [14], [12]. In extremely large chambers TLS is the only possibility [32]. A rather new piece of equipment is ZEB1 [34] and its enhanced version ZEB REVO [8]. These are handheld laser scanners utilizing the so called Simultaneous Localisation and Mapping (SLAM) technology [4], [5], [18]. Neglecting the price, this is probably the best tool for 3D cave mapping in general. It is easy to carry, easy to use, and the survey is extremely fast. It is not as accurate as TLS, but its accuracy is good enough to our needs, meeting grade XD according to the BCRA survey grading system, although it can be difficult to use in tight places. In [8] the authors tested accuracy on a planar wall and noticed 25-32 mm of deviation, operating at a range of 30 m. ZEB REVO has a weights of about 4kg, so it is heavier than a DistoX and has only IP64 rating, but the main problem is its price. In Hungary we can buy a ZEB1 for €20,000 and ZEB REVO is sold for €30,000. The SLAM software costs an another €13,000 , but you can also choose cloud processing on GeoSlam's servers and pay for each of your surveys.

Our new method, the Poor Man's Laser Scanner (PMLS) is a technique that makes it possible to acquire detailed and realistic models, almost like those obtained with tripod-based or handheld laser scanners, but using the surveying equipment that we already have, since most caving groups own, and are familiar with DistoX or similar devices. A PMLS model can also piece together disconnected 3D surveys created from other sources.

3　The proposed method

Our main contribution is a surface reconstruction method and associated software that applies the method. In our case we would like to create a realistic 3D model of a cave that corresponds to our measurements. If our measurements captures enough information about the geometric layout of the cave, then our model will correspond to reality. Our algorithm solves the problem of surface reconstruction reliably and it could be assembled from pieces of software already implemented by others, so we were able to realize the method with minimum software development efforts.

From our point of view a cave is a connected cavity. We try to approximately reconstruct the boundary surface of this cavity from rather sparsely sampled points in the form of a triangular mesh. As cavities are physical objects, the boundary of a cavity is a watertight surface. Watertight means that there are no holes in it. Such surfaces divide the 3D space into two parts: the interior and the exterior of the surface. The surface's interior is a solid – the cavity itself.

We have rather few samples of a complicated surface, so we must be able to use all the information that our measurements capture. In addition to the location of the splay shots there is also a relation between them. We know which shots were taken from the same station and the coordinates of all the stations are also known. We call a given station together with the splay shots measured from that station a **hedgehog**. So, we are looking for a watertight surface that satisfies two constraints:

Constraint 1. *The splay shots lie on the surface.*

Constraint 2. *The segments connecting the splay shots to their stations are in the interior of the surface.*

Unfortunately for finite number of splay shots there are infinitely many feasible surfaces, i.e. that satisfy the constraints, and most of them are very unrealistic. For instance, the surface that resulted from replacing the segments in the hedgehogs with poles satisfies the constraints but cannot be accepted as a cave model. It is clear, that we have to select the best, or at least a rather good surface from the feasible solutions. We formulate a constraint optimization problem. In such problems the solutions that satisfy the constraints are called *feasible* solutions. The goal is to find a feasible solution with optimal value of a function called the *objective function*.

The criterion according to which we choose a good surface, i.e. the objective function, will be the bending energy [15] [33], which is defined for the surface S as:

$$E_b(S) = \frac{1}{2}\int\limits_S H^2 dA \tag{1}$$

where H is the mean curvature [26], i.e. the sum of the principal curvatures $H = \kappa_1 + \kappa_2$ and dA is the differential area. A feasible surface with low bending energy will likely be free of unnecessary and undesirable "bending" and ""wrinkles"". Our algorithm consists of four steps. In the first three steps we construct an acceptable surface that satisfies the above constraints, at least for the vast majority of the splay shots. In the last step, we deform this surface to find a feasible solution with low bending energy.

Algorithms processing signals or measurements about real world phenomena usually have to incorporate the ability of detecting and removing outliers, i.e. anomalous measurements. Our algorithm also applies outlier detection. We remove the outliers, and do not require the constraints to be satisfied with respect to the outliers. An outlying splay shot can be the result of erroneous measurement with extremely long or short distance reading. If the laser beam

accidentally hits a drop of water, the distance reading can be excessively long. On the other hand short outliers usually result from shots on the surveyors own body or objects that should be skipped over, like ropes or other artificial equipment in the cave. Unfortunately outliers can also be caused by insufficient sampling (see section 3.2), so sometimes outlier detection can make bad decisions and remove accurate measurements, which leads to useful information being lost.

3.1 Extending the hedgehogs

In the first step for each station we try to find all such points that are likely to be visible from the given station but were measured from some other station. In other words for each splay shot we determine all the stations that it is visible from, and we add the point to the hedgehogs of these stations. We call the resulting hedgehogs the **extended hedgehogs**. In Figure 1 we can see the extended hedgehogs of 1275 splay shots measured from two stations. The survey took place in the Mátyás-hegyi cave under Budapest.
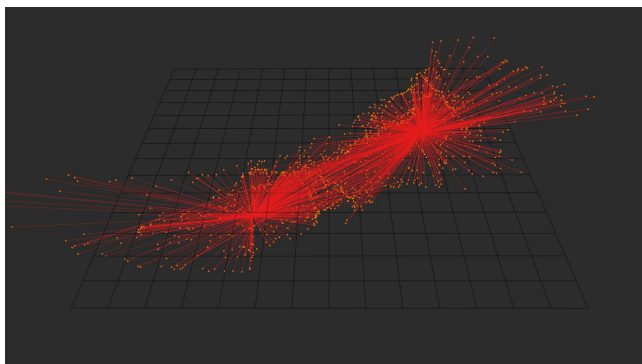

Fig. 1. Extended hedgehogs (Balázs Holl's survey)

3.2 Reconstructing surfaces for the extended hedgehogs separately

In this step we create a watertight surface for each extended hedgehog separately. All surfaces will be watertight and will satisfy our two constraints with respect to their own extended hedgehog. First, we cut back the splay shots to unit length, centered to the station (Figure 2). Second, we take the convex hull of the endpoints of the unit length shots. If the points are in general position, i.e. any four points are not coplanar, then the convex hulls will be a polyhedron with triangular faces (Figure 3) [25], [7]. Third, we keep the triangulation, i.e. the connectivity among the points, but put them back to their original positions (Figure 4). The resulting triangular surface is called the **turtle** of the given extended hedgehog. Note that turtles are watertight triangular surfaces, that are free of self-intersections, so they are polyhedrons and encapsulates three-dimensional solids. Each turtle estimates the part of the cave that is visible from
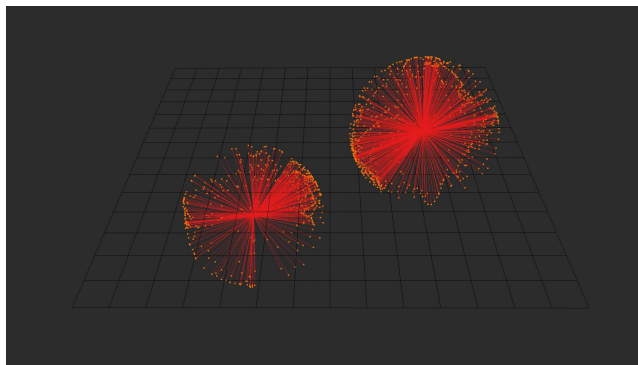

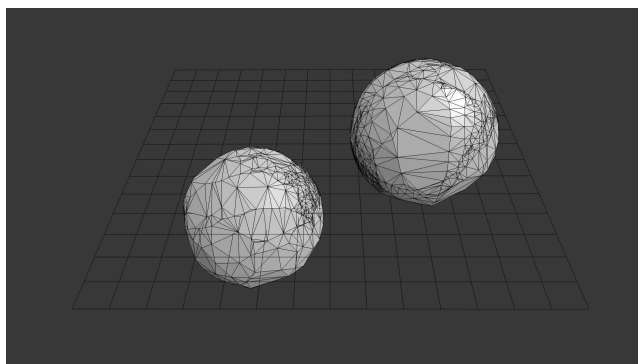Fig. 2. Extended hedgehogs with unit length splay shots
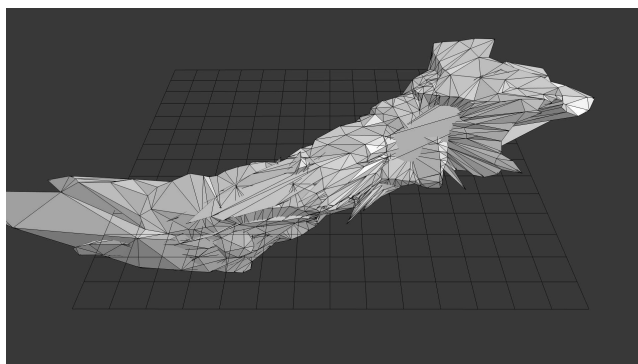

Fig. 3. Convex hull of the endpoints

Fig. 4. Overlapping turtles

its station with an interpolation of the distance readings in the spherical coordinate system centered at the given station.

We propose here two conditions on the samples that are necessary for the correctness of our algorithm.

Condition 3. *Neighbouring turtles are overlapping.*

Condition 4. *Splay shots that do not lie on the boundary of the volumetric union of the turtles, but in the interior of the union, come from erroneous measurements and they can be safely considered as outliers.*

We assume that neighbouring turtles are overlapping. This can easily be guaranteed by taking overlapping measurements from the corresponding stations. Condition 3 implies that neighbouring turtles are not disjoint and the resulting model will not be disconnected.

Condition 4 is necessary for correct outlier detection. The process of extending the hedgehogs in section 3.1 ensures that splay shots lying in the interior of any turtle do not exist. Turtles are star-shaped objects, i.e. there exists a point, the station, from which all the points lying in the interior or on the boundary of the turtle are visible. So the extending procedure has to result in turtles that do not contain any splay shots in their interior. Otherwise the given point should be considered visible from the station and added to the hedgehog of the station. In that case the given point becomes a vertex of the turtle. Unfortunately such boundary vertices may still lie in the interior of the union. To fulfil Condition 4 volumetric objects inside the cave, that are large enough to survey, like large stalactites or the bridge in Figure 7 have to be surveyed from at least two opposite sides.

3.3   Creating the union of extended turtles based on voxelisation

The union of the separate turtles are prepared by a robust method based on voxelization. Voxels are 3-dimensional pixels. Voxelisation means that we divide the space into many small cubes, just like digital images are built up from pixels. In this step we create a 3-dimensional binary image where each voxel represents the centre of a small cube. We set the value of a voxel to 1 if the centre-point is in the inside of any turtle, otherwise we set it to 0. We remesh (triangulate) the boundary of the volume made up of voxels with a value of 1. In Figure 5, we can see the resulting surface. The black dots shows the splay shots. By the remeshing we make the
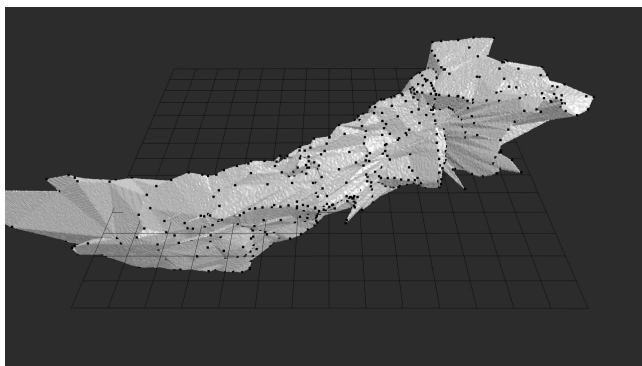

Fig. 5. Voxelized union of turtles

mesh rather dense, so it will have much more vertices than the number of the splay shots. We proceed in this way, because we will deform this mesh by moving its vertices while maintaining the connections, i.e. the triangles, among the vertices.

Under Condition 4 the theoretical union of the turtles satisfies Constraints 1 and 2 neglecting some erroneous measurements. The voxelised union is only an estimation of the true union and may lead to additional splay shots that dissatisfy the constraints. Voxelisation is robust because it introduces a simple form of *regularisation* since volumetric features with extremely small volume, like needles or blades will likely disappear. The corresponding samples will not satisfy the constraints, but usually they can be safely regarded as outliers resulting from measurement errors.

3.4   Optimization

During this step we shall deform the union in order to minimize the bending energy of the surface and conform to the constraints with as few "wrinkles" as possible. In a continuous model, deformation is a function p defined on the points of the surface to deform. For all points we assign its new position: $p : S \rightarrow S'$, where S and S' are sets of coordinate vectors. We assume that p is a regular parametrisation of the new surface with the old one, i.e. the mapping is one to one and continuously differentiable at least two times. Regular means that linearly independent directions remain independent during the mapping. We apply the method described in [20] to minimize the bending energy. For the sake of efficient computing we apply an approximation:

$$E_b\left(S'\right) \approx \frac{1}{2} \int\limits_{S} \langle \Delta p, \Delta p \rangle \, dA \qquad (2)$$

where Δ is the Laplace-Beltrami operator on the reference surface and $\langle, \rangle$ denotes the inner product in $\mathbb{R}^3$. The related Euler-Lagrange equation is the biharmonic equation with the unknown function p:

$$\Delta^2 p = 0 \qquad (3)$$

The approximation in (2) requires that p be closely isometric, which likely does not hold for large deformations. The integral in (2) is sometimes called the Laplacian energy. In addition to minimising (2), we have to ensure that our constraints are satisfied. Constraints can be incorporated into our framework as boundary values of the unknown function p in the biharmonic differential equation. For some points on the reference surface, i.e. the union, we can prescribe new positions by boundary values of the form:

$$p\left(x_i\right) = c_i \qquad (4)$$

where x_i will be some selected vertices of the reference mesh and c_i will be the new coordinates of these vertices.

We choose x_i and c_i as follows. For all vertices v_i of the reference mesh we assign the closest splay shot:

$$f\left(v_i\right) = \underset{c \in C}{\arg\min}\, d(v_i, c)$$

where C is the set of the sample points (splay shots) and d is the euclidean distance in the 3-dimensional space. For each

c_i from the range of f let

$$x_i = \arg\min_{v \in f^{-1}(c_i)} d(v, c_i)$$

where $f^{-1}(c_i) = \{v : f(v) = c_i\}$. The splay shots that were not assigned to any vertex by f are considered outliers (erroneous measurements) and removed.

Solving the above boundary value problem and evaluating the resulting function p in every vertex of the reference mesh gives the vertices of the deformed mesh. While moving the vertices to their new positions, we maintain the same triangulation. The deformation assures that constraint 1 will be satisfied with respect to the measurements that have not been regarded as outliers. Satisfying constraint 2, namely keeping the segments of the hedgehogs in the interior is achieved in a less elegant way. We detect the segments that have some interval outside the new surface. We sample these intervals equidistantly. For these new points we assign vertices from the original mesh in the same way as for the splay shots, and we add a further boundary value condition, but we keep the assigned vertices in their rest positions, i.e.

$$p(x_i) = x_i \tag{5}$$

We solve this extended boundary value problem again on the original union mesh. Figure 6 shows this solution in the case of our example. In Figure 7 we can see the same mesh
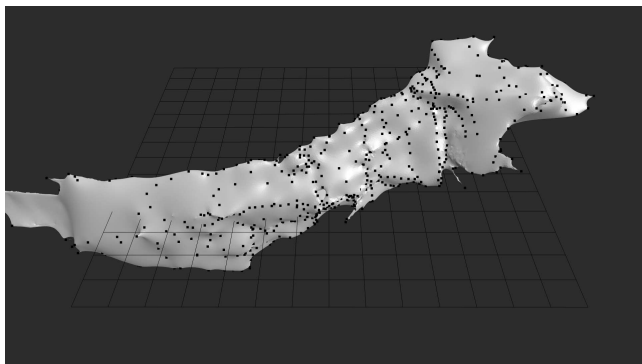

Fig. 6. Final biharmonic surface

from the inside (left) and a photo of the real cave from nearly the same point of view.

Note that our algorithm estimates cave geometry by interpolating instead of approximating the splay shots. This implies that we do not apply any error model in order to eliminate the effects of errors in the measurements that have not been removed by outlier detection. The algorithm is designed to work on extremely sparse samples, so interpolation seemed to be a more appropriate approach than approximation. In the case of having a large number of splay shots, it is worth to apply smoothing on the resulting mesh as a post processing step and achieve a suitable smooth approximation.

4 Implementation

As PMLS is a project done by hobby-cavers, we did not have much time for software development. It was critical to find an algorithm that can be assembled from already existing software tools with moderate programming efforts. Most of the tools we have applied are free and open source. The only commercial program that we used is Matlab [22]. Matlab is ideal for fast implementation of concepts and algorithms to verify, and to create a software prototype that can even be handled to the users for testing.

To solve the surface reconstruction problem we used several free and open source software in addition to Matlab. Considering the details of our algorithm, the step of creating the turtles from hedgehogs is done by Matlab functions. For extending the hedgehogs with points measured from other stations we used the fast ray casting software: opcode [30] through a modified Matlab wrapper [31]. The voxelized union is done by Iso2mesh [10], which is a package containing many mesh processing tools originating from the field of 3D medical imaging. The optimization of the bending energy is performed by the biharmonic deformation function of LibIgl [19].

In order to effectively try out a concept, the developer needs a tool for visualizing the results. The program that we used for that purpose was Blender [3]. Blender is a software package for creating 3D animations. Its coolest feature is that we can make a fly-through of the models. During my talk at the Eurospeleo conference I demonstrated the potential of PMLS by performing a fly-through inside one of our cave models [9]. Blender is excellent for viewing every tiny detail of 3D models. In addition, Blender's functionalities can be extended with add-ons written in the python programming language. On the other hand Matlab has a python interface, i.e. Matlab functions can be called from python. It was straightforward to create a graphical user interface for our method in the form of a Blender add-on.

We can load the input data from csv files, then we can view and edit the hedgehogs in Blender. For instance, we can delete erroneous measurements. The steps of the surface reconstruction process can be triggered by pushing buttons. At the end we can view the resulting mesh, and we can export it in many kinds of file formats. Our software will soon be available for download at cave3d.org.

5 Guidelines for on-site work

The geometric layout of caves can be very complicated with features at all scales. So do not try to make a perfect job, because it is impossible. Note that DistoX can have an error of 1-2 degrees in the horizontal angle, and try to capture details in a reasonable scale. The "take it easy" approach is more effective than being a perfectionist.

Fig. 7. The interior of the result (left), photo of the same place by András Hegedűs (right)

Typically we take about 50 to 500 splay shots from one station. Stations can be on the walls, as in conventional centreline survey, or a station can also be on a tripod. The only strict rule that the surveyor has to satisfy is to make sure that the measurements performed from neighbouring stations are heavily overlapping. You should take splay shots at least until the neighbouring stations, i.e. each turtle contains its neighbouring stations.

On the other hand it is wise to avoid long range shots if the given section can be surveyed from a closer station, because errors in the angle cause displacement of points, which are proportional to the distance. Try to think a bit in spherical coordinates. All your measurements assign a distance to a pair of angles. The first step of surface reconstruction will be an interpolation of the distance function, which should estimate well the cave section that is surveyed from the given station. The most important thing is to make measurements on corners and peaks, and to survey edge like features with some detail. Flat surfaces can be surveyed with a small number of shots. Avoid shots, where small changes in the direction can cause large errors in the distance. This is especially the case at some edges where a hidden surface

becomes visible (discontinuities of the distance function), or by measuring walls nearly parallel to the laser beam. Discontinuities have to be surveyed on both the near and the far sides, so do not skip such edges, but keep off a bit from the true edge.

If a volumetric object inside a cave is surveyed, there should be enough samples on its opposite sides to fulfill condition 4.

We found that it is convenient to use a tripod where it is possible. Balázs Holl built us a special 3-axis, non-magnetic tripod head as shown in (Figure 8) and in use in (Figure 9). Tripod based stations provide points of view in the interior of passages and chambers, which are usually much better than on-the-wall stations.

An upgrade to the Disto firmware by Beat Heeb accelerates the measuring process. It makes a scan mode available, so it is not necessary to push the button for each shot, but the device samples automatically as fast as it can. A shot requires one or two seconds based on the reflectivity of the wall.

We scanned the place with TLS from six positions what resulted a really high density pointcloud therefore it was necessary to resample the points in a 1 cm grid. Then the surface model was transformed (only translation and rotation, without scaling) into the coordinate system of the TLS point cloud using the Iterative Closest Point (ICP) algorithm.

We measured the distances of all the 24.5 millions samples in the mentioned grid points from the surface with the software CloudCompare. Figure 10 shows the orthog-
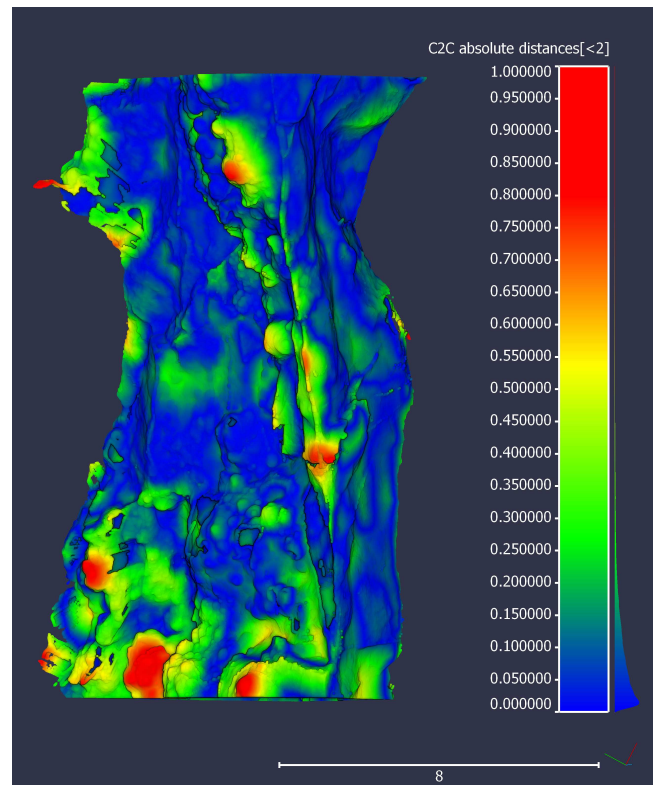

Fig. 8. Tripod-head for DistoX2

Fig. 9. Surveying in Bányász cave

Fig. 10. Deviation of point cloud samples

6 Results

6.1 Validation against a TLS survey

We need to validate the results to see how realistic the PMLS cave surveying technique is. For the validation we picked up an easily accessible place in the Mátyás-hegyi cave where also complex

and internal surfaces are found (see Figure 7). We acquired this place by Faro Focus 3D S 120 TLS (ranging error is +- 2mm at 10m) to validate the surface whose creation was shown in Section 3. As our models are very inaccurate on their endings, where the walls are visible only from a single station, we cut off the two ends, and kept only a 16.5 meter section around the bridge in Figure 7.

onal projection of the point cloud viewed from above and coloured according to its deviation from the model. Every point was closer to the model than one meter. The 92% of the samples are closer than 300mm and the 85% of the points are closer than 200mm. In Figure 10 we can see that errors larger than 500mm are due to small features that was not surveyed or was not even visible from the two stations of the PMLS survey. The cumulative histogram of the distances can be found in Figure 11. The mean of the distances is 113mm. Since the largest errors are caused by not surveying some narrow cracks, the median is a more appropriate descriptive measure, which is only 74mm. Considering that the 860 splay shots that lie on the analysed surface have a 52mm mean deviation from the TLS point cloud these results are very impressive, and show that our interpolation technique is rather effective.
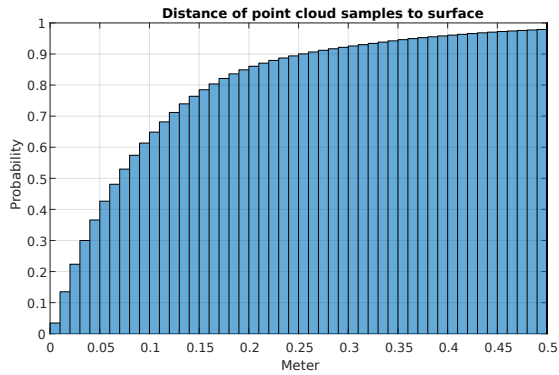
Fig. 11. Histogram of point cloud deviation

6.2 Bányász cave

Our main project is surveying the Bányász cave, the deepest cave in Hungary (273 m). The cave is a true pothole, which requires the heavy use of SRT. The entire cave is 830m long, from which we have surveyed 810m. Figure 12 shows the othogonal projection of the result. The on-site work took fifteen days. One surveying team have worked 4-8 hours on each day. Usually we have worked with a single disto, but there were four days when we could use two distos. We have measured about 61000 splay shots from 197 stations.

6.3 Legény cave

The surveyed section of Legény cave is not so long as in the case of Bányász cave. It is only 390 m, but the cave has a rather complicated geometric layout, see (Figure 13). The surveying took three days. On the first two days we used a single DistoX, while on the last day we could exploit two Distos. We measured more than 24,000 points from 112 stations.

Fig. 12. Bányász cave

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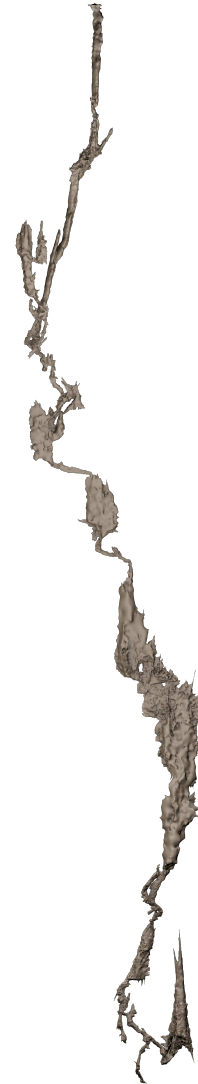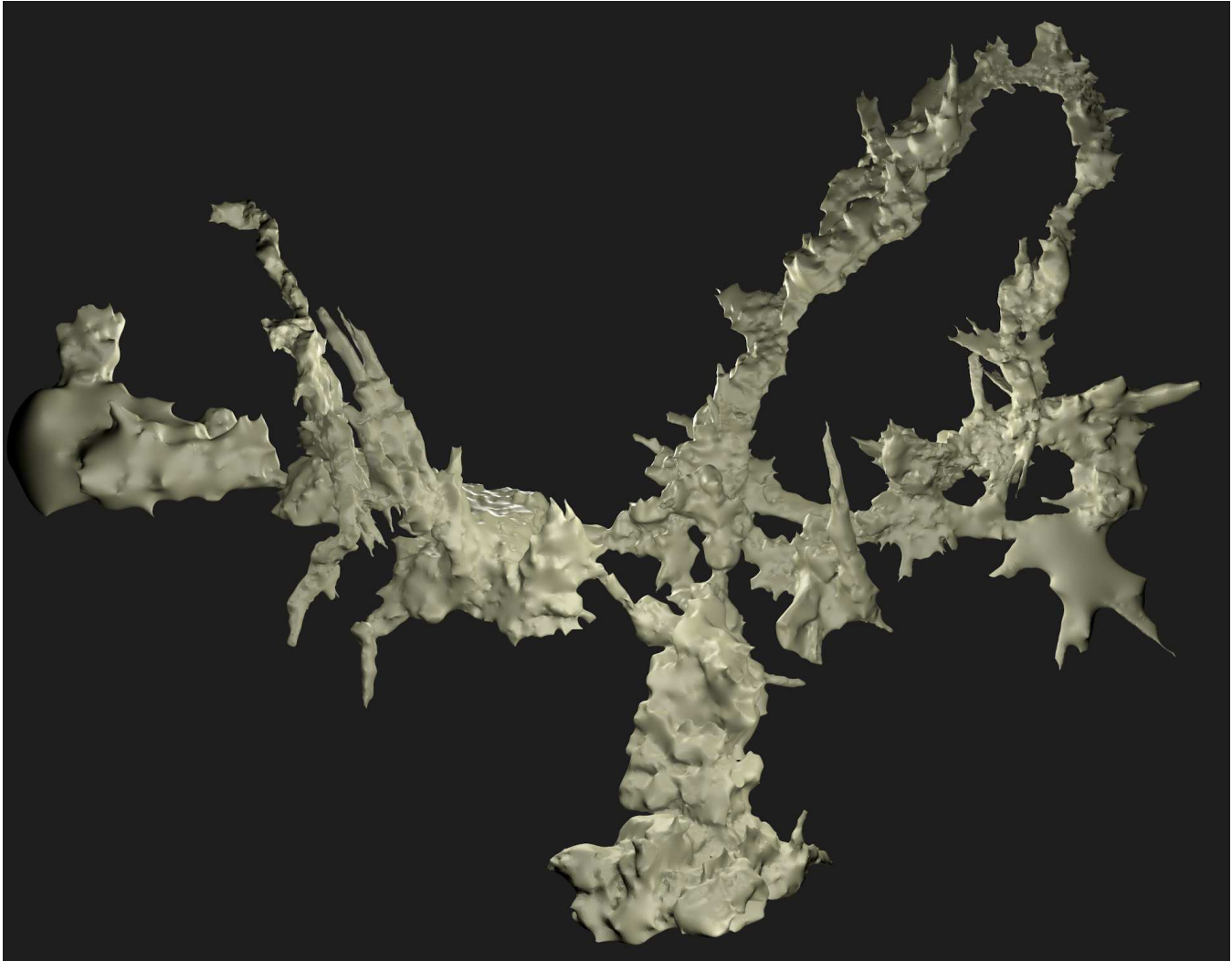
Fig. 13. Legény cave

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